Click Here

Difference Between Linear and Non-Linear Equations: While solving mathematical problems, you may have seen types of equations. Few Equations can contain only numbers, others consist of only variables while some consist of both numbers and variables. Linear and nonlinear equations usually consist of numbers as well as variables. In this article, we will understand the difference between Linear and Non-Linear Equations along with sample problems. What are Linear and Non-Linear Equations?Before starting with the difference between linear and non-linear equations, let us first understand the definition of Linear and Non-Linear equations. Linear EquationA linear equation is such that it forms a straight line. Linear means something related to a line. All the linear equations are used to construct a line. Linear Equations are conditions of the principal request. These conditions are characterized by lines in the arranged framework. An equation for a straight line is known as a Linear equation. The overall portrayal of the straight-line condition is y=mx+b, where m is the slant of the line and b is the y-catch. Non-Linear EquationA non-linear equation is such that it does not form a straight line. It looks like a curve in a graph and has a variable slope value. A non-linear equation is generally given by ax2+by2 = c. where x and y are variables while a,b, and c are constant values. Difference Between Linear and Non-Linear EquationsThe major difference between linear and nonlinear equations is given here for the students to understand it more naturally. The differences are provided in a tabular form with examples: S No. Linear EquationsNon-Linear Equations1.It forms a straight line or represents the equation for the straight lineIt does not form a straight line but forms a curve.2.It has only one degree. We can also define it as an equation having a maximum degree of 1.A nonlinear equation has the degree as 2 or more than 2, but not less than 2.3.All these equations form a straight line in the XY plane. These lines can be extended in any direction but in a straight form.It forms a curve and if we increase the value of the degree, the curvature of the graph increases.4.The general representation of a linear equation is; y = mx +c Where x and y are the variables, m is the slope of the line and c is a constant value. The general representation of nonlinear equations is; ax2 + by2 = c Where x and y are the variables and a, b and c are the constant values 5.Linear Equations are much simpler to solve.Non-linear Equations are tricky in nature.6.Linear Equations are time saving.Non-linear equations are time-consuming.7.Examples: 10x = 19y + x + 2 = 04y = 3x99x + 12 = 23 yExamples: x2+y2 = 1x2 + 12xy + y2 = 0x2+x+2 = 25Sample Problems - Difference Between Linear and Non-Linear EquationsProblem 1: Solve the linear equation 3x+18 = 2x + 21. Solution: Given, 3x+18 = 2x + 21 ⇒ 3x – 2x = 21 – 18 ⇒ x = 3 Problem 2: Solve x = 12(x +2) Solution: x = 12(x + 2) x = 12x + 24 Subtract 24 from each side x – 24 = 12x + 24 – 24 x – 24 = 12x Simplify 11x = -24 Isolate x, by dividing each side by 11 11x / 11 = -24/11 x = -24/11 Problem 3: Solve the nonlinear equation x+4y = 1 and x = y. Solution: Given, x+4y = 1 x = y By putting the value of x in the first equation we get, ⇒ y + 4y = 1 ⇒ 4y = 1 ⇒ y = 1/4 ∴ x = y = 1/4 Problem 4: Example: Solve the nonlinear equation x+2y = 1 and x = 2 Solution: Given, x+2y = 1 x = 2 By putting the value of x in the first equation we get, ⇒ 2+ 2y = 1 ⇒ 2y = -1 ⇒ y = -1/2 ∴ y=-1/2 Definition and Characteristics Linear equations are characterized by their consistent slope, which results in a straight line when plotted on a graph [1:1]. They can be expressed in the form "y = mx + b", where the relationship between variables is proportional and additive. For example, in the equation "y = 3x", the output (y) is always three times the input (x), regardless of the value of x [1:2]. Nonlinear equations, on the other hand, involve variables raised to powers or multiplied together, resulting in curves rather than straight lines on a graph [1:3]. Linear vs. Nonlinear Differential Equations In the context of differential equations, linear ODEs can be expressed as a sum of derivatives multiplied by functions of independent variables [2:4]. Nonlinear ODEs include terms that are functions of dependent variables, such as absolute values or products of variables [3:1]. This distinction affects the behavior and solution methods for these equations. Visualization Differences While the visualization methods for linear and nonlinear ODEs are similar, the resulting visuals differ due to the nature of the functions involved [4:1]. Linear systems tend to produce uniform slope fields, whereas nonlinear systems can exhibit more complex patterns and behaviors. Proportionality and Constants The concept of proportionality is related to linear equations. When converting a proportional relationship to an equality, a constant is introduced to account for additional factors, such as time in the equation "Distance = Time x Speed" [5:1][5:5]. This constant ensures the equation accurately represents the relationship between the variables under specific conditions. The main difference between linear and nonlinear differential equations lies in the degree of the equation and the complexity of their solutions. Linear Differential Equations: Have a maximum degree of one. Form a straight line when graphed. Can be written in the form of $$y' + p(x)y = q(x)$$ or $$y'' + p(x)y' + q(x)y = r(x)$$. Examples: $$y' + 2xy = 0$$, $$y'' + 2x' + y = 0$$. Nonlinear Differential Equations: Have a degree of 2 or more. Form a curve when graphed. Cannot be written in the form of linear differential equations. Examples: $$y' + \frac{1}{x} = 0$$, $$y' + x^2y = 0$$. Understanding the difference between linear and nonlinear equations is essential in mathematics and physics. Linear equations are generally considered "simple" and can be solved using well-developed theories, while nonlinear equations are considered "complicated" and their solutions are more challenging to find.The main difference between linear and nonlinear differential equations lies in the presence or absence of products of the unknown function (y) and its derivatives in the equation. Here is a table summarizing the differences between linear and nonlinear differential equations: Feature Linear Differential Equations Nonlinear Differential Equations Definition A linear differential equation does not contain any products of y and its derivatives A nonlinear differential equation contains products of y and its derivatives Form Linear differential equations are usually written in the form: $$\alpha \frac{dy}{dx} + \beta y + \gamma = 0$$, where α, β, and γ do not depend on y. Nonlinear differential equations do not have a standard form and can contain any powers of the unknown function or its derivatives (except for 0 or 1), or even a nonlinear combination of y and its derivatives. Solution Most linear differential equations have solutions that are made of exponential functions or integrals of explicit functions. This is not true for nonlinear equations. It is more challenging to find mathematically exact solutions for nonlinear differential equations. They often require numerical methods or software to solve them. Examples $$\frac{dy}{dx} + y - x = 0$$ (a linear first-order differential equation). $$\frac{dy}{dx} = y^2$$ (a nonlinear first-order differential equation). Understanding the difference between linear and nonlinear differential equations is important because linear differential equations are generally easier to solve and solve analytically, while nonlinear differential equations often require numerical methods or software to solve them. Linear differential equations are equations in which the dependent variable and its derivatives appear linearly. This means that the highest power of the dependent variable or its derivatives is 1. Nonlinear differential equations, on the other hand, are equations in which the dependent variable and its derivatives appear nonlinearly. This means that the highest power of the dependent variable or its derivatives is greater than 1 or that they are multiplied together. Linear differential equations are generally easier to solve analytically and have well-defined solutions, while nonlinear differential equations often require numerical methods or approximation techniques to find solutions. Additionally, linear differential equations have the property of superposition, meaning that if two solutions are found, their linear combination is also a solution. Nonlinear differential equations do not possess this property. Differential equations are fundamental mathematical tools used to describe various phenomena in science, engineering, and economics. They are equations that involve derivatives and are classified into different types based on their properties. Two major categories of differential equations are linear and nonlinear differential equations. In this article, we will explore the attributes of linear and nonlinear differential equations, highlighting their differences and applications.Linear Differential EquationsLinear differential equations are equations where the dependent variable and its derivatives appear linearly. In other words, the highest power of the dependent variable or its derivatives is 1. These equations can be written in the form:an(x)y(n) + an-1(x)y(n-1) + ... + a1(x)y' + a0(x)y = f(x)wherey is the dependent variable,x is the independent variable,n is the order of the differential equation, andai(x) andf(x) are functions ofx.One of the key attributes of linear differential equations is that they satisfy the superposition principle. This means that ify1(x) andy2(x) are solutions to the linear differential equation, then any linear combination of them, such asC1y1(x) + C2y2(x), whereC1 andC2 are constants, is also a solution.Linear differential equations have well-defined and predictable behavior. Their solutions can often be obtained analytically using various techniques such as separation of variables, integrating factors, or Laplace transforms. This makes them particularly useful in many areas of science and engineering, including physics, electrical circuits, and control systems.Nonlinear Differential EquationsNonlinear differential equations are equations where the dependent variable or its derivatives appear nonlinearly. This means that the highest power of the dependent variable or its derivatives is greater than 1, or they are multiplied or divided by each other. Nonlinear differential equations can take various forms and are generally more challenging to solve analytically compared to linear differential equations.Unlike linear differential equations, nonlinear differential equations do not satisfy the superposition principle. This means that finding solutions to nonlinear equations requires different techniques and approaches. In many cases, numerical methods or approximation techniques are employed to obtain approximate solutions.Nonlinear differential equations are prevalent in many fields, including biology, chemistry, economics, and population dynamics. They are often used to model complex systems where linear approximations are insufficient. Nonlinear equations can exhibit rich and intricate behavior, including chaos, bifurcations, and limit cycles, making them a powerful tool for studying complex phenomena.ComparisonNow that we have explored the attributes of linear and nonlinear differential equations, let's compare them in terms of various aspects:1. LinearityAs mentioned earlier, linear differential equations have a linear relationship between the dependent variable and its derivatives. Nonlinear differential equations, on the other hand, have a nonlinear relationship. This fundamental difference in linearity affects the behavior and solutions of the equations.2. Superposition PrincipleLinear differential equations satisfy the superposition principle, allowing for the combination of solutions to obtain new solutions. Nonlinear differential equations do not satisfy this principle, making their solutions more challenging to find and analyze.3. SolvabilityLinear differential equations often have closed-form solutions that can be obtained analytically using various techniques. Nonlinear differential equations, on the other hand, are generally more difficult to solve analytically and often require numerical or approximation methods to obtain solutions.4. StabilityLinear differential equations have well-defined stability properties. The behavior of their solutions can be classified as stable, unstable, or marginally stable based on the eigenvalues of the associated linear operator. Nonlinear differential equations, on the other hand, can exhibit a wide range of stability behaviors, including stable equilibrium points, limit cycles, and chaotic behavior.5. ApplicationsLinear differential equations find widespread applications in various fields, including physics, engineering, and control systems. They are particularly useful in modeling systems with linear relationships and predictable behavior. Nonlinear differential equations, on the other hand, are essential for modeling complex systems with nonlinear interactions, such as biological systems, chemical reactions, and economic dynamics.ConclusionLinear and nonlinear differential equations are two distinct categories of equations with different properties and applications. Linear differential equations have a linear relationship between the dependent variable and its derivatives, satisfy the superposition principle, and often have closed-form solutions. Nonlinear differential equations, on the other hand, have a nonlinear relationship, do not satisfy the superposition principle, and are generally more challenging to solve analytically. They find applications in modeling complex systems with nonlinear interactions. Both types of equations play a crucial role in understanding and describing various phenomena in science, engineering, and economics. Comparisons may contain inaccurate information about people, places, or facts. Please report any issues. Unit I: First Order Differential Equations « Previous | Next » Session Overview This session consists of an imaginary dialog written by Prof. Haynes Miller and performed in his 18.03 class in spring 2010. It takes the form of a debate between Linn E. R. representing linear first order ODE's and Chao S. doing the same for first order nonlinear ODE'. Session Activities Read the course notes: Linear vs. Nonlinear (PDF) Check Yourself Complete the problem set: Problem Set Part II Problems (PDF) Problem Set Part II Solutions (PDF) (Note: There is no Problem Set Part I in this session). « Previous | Next » I'm just starting to learn about ordinary differential equation and I'm still don't know how to find the difference between linear and non linear differential equation. I'm really confused about it, even after I'm reading my textbook. Would someone help me please? thank you very much Last edited: Oct 28, 2008 mathwonk its pretty much the difference between ones you can solve and ones you cannot. HallsofIvy Oh, now that's just silly! (Though pretty much true!) fabianz, do you know the difference between linear and nonlinear functions? A linear function must be of theform f(x)= ax+ b. Nothing more complicated than multiplying by a number and adding a number. For a function of several variables, "a" and "b" can be any function of the other variables and still be "linear in x". Any thing other than that is "non-linear". A differential equation if "linear" if it does not involve any non-linear functions of the dependent variable In order to numerically solve the ordinary differential equation \(\frac{d y}{d t}=-y\) for t > 0, with an initial condition y(0) = 1, the following scheme is employed \(\frac{y_{n+1}-y_{n}}{Δ t}=\frac{1}{2}\left(y_{n+1}+y_{n}\right)\) Here, Δt is the time step and yn = y(nΔt) for n = 0, 1, 2, ... This numerical scheme will yield a solution with non-physical oscillations for Δt > h. The value of h is 1\(\frac{3}{2}\)\(\frac{1}{2}\)2 India's Super Teachers for all govt. exams Under One Roof Enroll For Free Now Explanation: Given that, \(\frac{y_{n+1}-y_{n}}{\Delta t}=\frac{1}{2}\left(y_{n+1}+y_{n}\right)\) \(y_{n+1}-y_{n}=\frac{\Delta t}{2}\left(y_{n+1}+y_{n}\right)\) \(y_{n+1}-\frac{Δ t}{2} y_{n+1}=y_{n}+\frac{Δ t}{2} y_{n}\) \(y_{n+1}=\frac{2+Δ t}{2-Δ t} y_{n}\) For stability: \(\left|\frac{2+\Delta t}{2-\Delta t}\right| \leq 1\) ⇒ \(-1 \leq \frac{2+\Delta t}{2-\Delta t} \leq 1\) ⇒ \(-1 \leq-1+\frac{4}{2-\Delta t} \leq 1\) ⇒ \(0 \leq \frac{4}{2-\Delta t} \leq 2\) ⇒ \(\frac{1}{2} \leq \frac{2-\Delta t}{4} \leq 0\) ⇒ 2 ≤ 2 - Δt ≤ 0 ⇒ 0 ≤ - Δt ≤ -2 ⇒ Δt ≥ 2 India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students For the following partial differential equation, \(\rm x \frac{\partial^{2} f}{\partial x^{2}}+y \frac{\partial^{2} f}{\partial y^{2}}=\frac{x^{2}+y^{2}}{2}\) Which of the following option(s) is/are CORRECT? hyperbolic for x < 0 and y > 0elliptic for x > 0 and y > 0elliptic for x = 0 and y > 0parabolic for x > 0 and y > 0 Explanation: The most general case of second-order partial differential equation \(\rm A \frac{\partial^{2} u}{\partial x}+B \frac{\partial^{2} u}{\partial x \partial y}+C \frac{\partial^{2} u}{\partial y^{2}}+D \frac{\partial u}{\partial x}+E \frac{\partial u}{\partial y}+f u=G\) ...(i) Given that the partial differential equation is \(\rm x \frac{\partial^{2} f}{\partial x^{2}}+y \frac{\partial^{2} f}{\partial y^{2}}=\frac{x^{2}+y^{2}}{2}\) ...(ii) Now, Comparing (i) and (ii) A = x and B = 0, C = y Discriminant = B2 – 4AC = 0 – 4 × x × y = –4xy –4yx > 0 if x < 0 and y > 0 So, the given equation is hyperbolic if x < 0 and y > 0. Similarly, we can prove if x > 0, y > 0 given partial differential equation is elliptic also. India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students Which of the following is/are eigenvalue(s) of the Sturm–Liouville problem y'' + λy = 0, 0 ≤ x ≤ π, y(0) = y'(0), y(π) = y'(π)? Explanation: y'' + λy = 0 If λ > 0 say λ = k2 then y'' + k2y = 0 Auxiliary equation m2 + k2 = 0 m2 = ± ki General solution is y = a cos kx + b sin kx So, y' = - ak sin kx + bk cos kx Using y(0) = y'(0) ⇒ a = bk....(i) y(π) = y'(π) ⇒ a cos(kπ) + b sin kπ = - ak sin kπ + bk cos kπ ⇒ bk cos(kπ) + b sin kπ = - bk2 sin kπ + bk cos kπ ⇒ b(1 + k2)sin kπ = 0 ⇒ sin kπ = 0 ⇒ kπ = nπ, n = 1, 2, 3, ... ⇒ k = n, n = 1, 2, 3, ... Hence λ = n2, n = 1, 2, 3, ... So, λ = 1, 4, 9, ... Option (1) and (4) are true. India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students Let y(x) be the solution of the following initial value problem \(x^2 \frac{d^2 y}{d x^2}-4 x \frac{d y}{d x}+6 y=0\), x > 0, y(2) = 0, \(\frac{d y}{d x}(2)=4 .\) Then y(4) = _____. Answer (Detailed Solution Below) 32 Explanation: \(x^2 \frac{d^2 y}{d x^2}-4 x \frac{d y}{d x}+6 y=0\) Put \(x \frac{d y}{d x} = D'\) and \(x^2 \frac{d^2 y}{d x^2} = D'(D'-1)\) Then we get, \((D'(D'-1)-4D'+6)y=0\) Auxillary equation: \(m^2-5m +6=0\) ⇒ (m-3)(m-2) =0 ⇒ m= 2,3 \(y= c_1x^2 +c_2 x^3\) ⇒\(\frac{d y}{d x}= 2c_1x+3c_2x^2\) We are given y(2) = 0, \(\frac{d y}{d x}(2)=4 .\) as y(2) = 0 then \(4c_1+8c_2=0\) \(\frac{d y}{d x}(2)=4\) ⇒ \(4c_1+ 12c_2 =4\) ⇒ \(c_2=1\) and \(c_1=-2\) thus, \(y= -2x^2+x^3\) ⇒ y(4) = -32+64 = 32 India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students If the ordinary differential equation \(rm x^2\frac{d^2\phi}{dx^2}+x\frac{d\phi}{dx}+x^2\phi=0\), x > 0 has a solution of the form ϕ(x) = xr \(\Sigma_{n=0}^\infty a_nx^n, \) where an's are constants and a0 ≠ 0, then the value of r2 + 1 is _____. Answer (Detailed Solution Below) 1 dϕdx=rxr−1∑n=0∞anxn+xr∑n=1∞nanxn−1. \frac{d\phi}{dx} = r x^{r-1} \sum_{n=0}^\infty a_n x^n + x^r \sum_{n=1}^\infty n a_n x^{n-1}." id="MathJax-Element-61-Frame" role="presentation" style="text-align: center; position: relative;" tabindex="0"> dϕdx=rxr−1∑n=0∞anxn+xr∑n=1∞nanxn−1. \frac{d\phi}{dx} = r x^{r-1} \sum_{n=0}^\infty a_n x^n + x^r \sum_{n=1}^\infty n a_n x^{n-1}." id="MathJax-Element-61-Frame" role="presentation" style="text-align: center; position: relative;" tabindex="0">dϕdx=rxr−1∑n=0∞anxn+xr∑n=1∞nanxn−1.Unknown node type: span" id="MathJax-Element-91-Frame" role="presentation" style="text-align: center; position: relative;" tabindex="0">dϕdx=rxr−1∑n=0∞anxn+xr∑n=1∞nanxn−1.Unknown node type: span dϕdx=rxr−1∑n=0∞anxn+xr∑n=1∞nanxn−1. \frac{d\phi}{dx} = r x^{r-1} \sum_{n=0}^\infty a_n x^n + x^r \sum_{n=1}^\infty n a_n x^{n-1}." id="MathJax-Element-61-Frame" role="presentation" style="text-align: center; position: relative;" tabindex="0">India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students Particular integral of \(\frac{{{d^2}y}}{{d{x^2}}} + 3\frac{{dy}}{{dx}} + 2y = {e^{ - 2x}} + \sin x\) is \(x{e^{ - 2x}} + \frac{1}{{10}}\left({\sin x - 3\cos x} \right)\)\(- x{e^{ - 2x}} + \frac{1}{{10}}\left({\cos x - 3\sin x} \right)\)\(- x{e^{ - 2x}} + \frac{1}{{10}}\left({\sin x + 3\cos x} \right)\) Option 2 : \(- x{e^{ - 2x}} + \frac{1}{{10}}\left({\sin x - 3\cos x} \right)\) Concept: When Φ(x) = eax + V(x) \(\{{\rm{P}}.{\rm{I}} = \frac{1}{{{\rm{f}}\left({\rm{D}} \right)}}\left\{ {{{\rm{e}}^{ax}} + {\rm{\;V}}\left({\rm{x}} \right)} \right\}\ \Rightarrow \frac{1}{{{\rm{f}}\left({\rm{D}} \right)}}{{\rm{e}}^{{\rm{ax}}}} + \frac{1}{{{\rm{f}}\left({\rm{D}} \right)}}{\rm{V}}\left({\rm{x}} \right)\) \({\rm{P}}.{\rm{I}} = \frac{1}{{{\rm{f}}\left({\rm{D}} \right)}}{{\rm{e}}^{{\rm{ax}}}} = \frac{1}{{{\rm{f}}\left({\rm{a}} \right)}}{{\rm{e}}^{{\rm{ax}}}} = {\rm{x}}\frac{1}{{{\rm{f'}}\left({\rm{a}} \right)}}{{\rm{e}}^{{\rm{ax}}}}\) Calculation: \(\frac{{{d^2}y}}{{dx^2}} + 3\frac{{dy}}{{dx}} + 2y = {e^{ - 2x}} + \sin x\) \(\therefore {\rm P}.{\rm I} = \frac{1}{{{\rm D}^2 + 3D + 2}}{e^{ - 2x}}\) \(\Rightarrow x\frac{1}{{2D + 3}}{e^{ - 2x}\;}\;\left\{ {\because f\left({ - 2} \right) = 0} \right\}\) \(\Rightarrow x\frac{1}{{2\left({ - 2} \right) + 3}}{e^{ - 2x}}\) ∴ -xe-2x \(\frac{1}{{{D^2} + 3D + 2}}\sin x\) \(\Rightarrow \frac{1}{{ - \left({{1^2}} \right) + 3D + 2}}\sin x\) \(\Rightarrow \frac{1}{{3D + 1}}\sin x\) \(\Rightarrow \frac{{3D - 1}}{{9{D^2} - 1}}\sin x\) \(\Rightarrow \frac{{3D - 1}}{{9\left({ - {1^2}} \right) - 1}}\sin x\) \(\Rightarrow - \frac{1}{{10}}\left({3D - 1} \right)\sin x\) \(\Rightarrow - \frac{1}{{10}}\left({3\cos x - \sin x} \right)\) \(\Rightarrow \frac{1}{{10}}\left({\sin x - 3\cos x} \right)\) \(\therefore P.I = - x{e^{ - 2x}} + \frac{1}{{10}}\left({\sin x - 3\cos x} \right)\) India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students The ordinary differential equation \(\frac{{d^2u}}{{d{x^2}}} - 2{x^2}u + \sin x = 0\) is linear and homogeneouslinear and nonhomogeneousnonlinear and homogeneousnonlinear and nonhomogeneous Option 2 : linear and nonhomogeneous Concept: Identification of Non-linear Differential Equation : Ordinary Differential Equation Partial Differential Equation 1) The degree is more than 1. 1) The degree is more than 1. 2) The exponent of the dependent variable is more than 1. 2) The exponent of the dependent variable is more than 1. 3) The exponent of any derivative > 1. 3) The exponent of any derivative > 1. 4) Product of dependent variable with its any derivative is present. 4) Product of dependent variable with its any derivative is present. 5) Product of any two partial derivatives is present If any differential equation consists at least one of the above properties, then it is called non-linear differential equation and if any differential equation is free from all the above properties, then it is a linear differential equation. Given: \(\frac{{{d^2}u}}{{d{x^2}}} - 2{x^2}u + \sin x = 0\) It is free from all the four characteristics described in the table. Hence it is a linear differential equation. Product of dependent and independent variable i.e. x and u is present. Hence it is a non-homogenous equation. India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students The particular integral of (D + 3)2 y = 5x - log 2, is \(\frac{{{{\rm{5}}^{\rm{x}}}}}{{{\rm{3 + log5}}}}\, - \,\frac{{\log \,2}}{9}\)\(\frac{{{{\rm{5}}^{\rm{x}}}}}{{{{\left({{\rm{3 + log5}}} \right)}^2}}}\, - \,\frac{{\log \,2}}{9}\)\(\frac{{{{\rm{5}}^{\rm{x}}}}}{{{{\left({{\rm{3 - log5}}} \right)}^2}}}\, - \,\frac{{\log \,2}}{9}\) \(\frac{{{{\rm{5}}^{\rm{x}}}}}{{{{\left({{\rm{3 - log5}}} \right)}^2}}}\, + \,\frac{{\log \,2}}{9}\) Option 2 : \(\frac{{{{\rm{5}}^{\rm{x}}}}}{{{{\left({{\rm{3 + log5}}} \right)}^2}}}\, - \,\frac{{\log \,2}}{9}\) Given: (D + 3)2y = 5x - log 2 Formula Used: \(\frac{1}{f(D)}\ e^{ax} = \frac{1}{f(a)} \ e^{ax}\) e log x = x Calculation: We have, ⇒ (D + 3)2y = 5x - log 2 Particular Integral of the differential equation is = \(\frac{e^{log\ 5^x} - \log2. \ e^{0x}}{(D\ +\ 3)^2}\) = \(\frac{e^{xlog5}} {(D \ + \ 3)^2} - \frac{log 2}{ (0 + 3)^2}\) = \(\frac{e^{x \ log5}}{(3 + log 5)^2} - \frac{log2}{9}\) ⇒ \(\frac{5^x}{(3 \ + \ log5)^2} - \frac{log2}{9}\) ∴ The particular integral of (D + 3)2y = 5x - log 2 is \(\frac{5^x}{(3 \ + \ log5)^2} - \frac{log2}{9}\) India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students A differential equation is given as: \({x^2}\frac{{{d^2}y}}{{d{x^2}}} - 2x\frac{dy}}{{dx}} + 2y = 4\) The solution of the differential equation in terms of arbitrary constants C1 and C2 is y = C2x2 + C1x + 2\(y = \frac{{{C_1}}}{{{x^2}}} + {C_2}x + 2\)y = C1x2 + C2x + 4\(y = \frac{{{C_1}}}{{{x^2}}} + {C_2}x + 4\) Option 1 : y = C2x2 + C1x + 2 Given differential equation: \({x^2}\frac{{{d^2}y}}{{d{x^2}}} - 2x\frac{dy}}{{dx}} + 2y = 4\) is the standard form of Euler-Cauchy DE. So, let x = ez ⇒ dx = ezdz = x.dz \(\Rightarrow \frac{1}{{dz}} = \frac{x}{{dx}}i.e.\frac{d}{{dz}} = x\frac{d}{{dx}}\) Similarly, we can obtain \({x^2}\frac{{{d^2}}}{{d{x^2}}} = \frac{{{d^2}}}{{d{z^2}}} - \frac{d}{{dz}}\) ⇒ xD = θ ⇒ x2D2 = θ (θ - 1) Making these substitutions in given DE, we get \({x^2}\frac{{{d^2}y}}{{d{x^2}}} - 2x\frac{dy}}{{dx}} + 2y = 4\) (θ (θ - 1) – 2θ + 2) y = 4 θ (θ - 1) – 2(θ – 1) = 0 ⇒ Solution for this will constitute of CF & PI. ⇒ CF = (θ – 1) (θ – 2) = 0 ⇒ θ = 1 & θ = 2 ⇒ y = c1ez + c2e2z = c1x + c2x2 \(also,\;PI = \frac{1}{{f\left(\theta \right)}}{e^{az}} = \frac{1}{{f\left(a \right)}}{e^{az}} \Rightarrow \frac{{4 \times {e^{0z}}}}{{\left({\theta - 1} \right)\left({\theta - 2} \right)}} = \frac{{4 \times {e^{0z}}}}{{\left({ - 1} \right)\left({ - 2} \right)}} = \frac{4}{2} = 2\) ⇒ y = CF + PI = c2x2 + c1x + 2 {We can also solve this problem by differentiating option also, if we don't remember the process.} India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students The respective expressions for complementary function and particular integral part of the solution of the differential equation \(\frac{{{d^4}y}}{{d{x^4}}} + 3\frac{{{d^2}y}}{{d{x^2}}} = 108{x^2}\) are c1 + c2x + c3 sin√3x + c4cos√3x and 3x4 - 12x2 + cc2x + c3 sin√3x + c4cos√3x and 5x4 - 12x2 + cc1 + c3 sin√3x + c4cos√3x and 3x4 - 12x2 + cc1 + c2x + c3 sin√3x + c4cos√3x and 5x4 - 12x2 + c Option 1 : c1 + c2x + c3 sin√3x + c4cos√3x and 3x4 - 12x2 + c Concept: The complete solution of differential equation (D.E) is given by Y = C.F + P.I where, C.F = Complementary Function and P.I = particular Integral \(D = \frac{d}{dx}\) Calculation: Given: \(\frac{{{d^4}y}}{{d{x^4}}} + 3\frac{{{d^2}y}}{{d{x^2}}} = 108{x^2}\) where, \(D = \frac{d}{{dx}}\) \(\left({{D^4} + 3{D^2}} \right)y = 108{x^2}\) A.E = Auxilary eqaution is given by m4 + 3m2 = 0 m2(m2 + 3) = 0 \(\begin{array}{l} m = 0,\;0,\; \pm \sqrt 3 i;\\ \therefore CF = \left({{C_1} + {C_2}x} \right) + {C_3}\sin \left({\sqrt 3 x} \right) + {C_4}\cos \left({\sqrt 3 x} \right) \end{array}\) \(\begin{array}{l} PI = \frac{1}{{{D^4} + 3{D^2}}}\left({108{x^2}} \right)\\ = \frac{1}{{{3{D^2}\left[{1 + \frac{{{D^2}}}{3}} \right]}}\left({108{x^2}} \right)\\ = \frac{{36}}{{{D^2}}}\left[{1 + \frac{{{D^2}}}{3}} \right]^{ - 1}}\left({{x^2}} \right)\\ = \frac{{36}}{{{D^2}}}\times \left[{1 - \frac{{{D^2}}}{3} + \ldots } \right]\left({{x^2}} \right)\\ = \frac{{36}}{{{D^2}}}\left[{{x^2} - \frac{1}{3}\left(2 \right) + 0} \right]\\ = \int\!\!\!\int \left({36{x^2} - \frac{2}{3}} \right)dxdx\\ = 36\left({\frac{{{x^4}}}{{\left(4 \right)\left(3 \right)}} - \frac{2}{3}\frac{{{x^2}}}{{\left(2 \right)\left(1 \right)}}} \right)\\ = 3{x^4} - 12{x^4} \end{array}\) Hence, \(\begin{array}{l} CF = \left({{C_1} + {C_2}x} \right) + {C_3}\sin \left({\sqrt 3 x} \right) + {C_4}\cos \left({\sqrt 3 x} \right) \end{array}\) \(\begin{array}{l} P.I=(3{x^4} - 12{x^4}) \end{array}\) \(\begin{array}{l}Y = \left({{C_1} + {C_2}x} \right) + {C_3}\sin \left({\sqrt 3 x} \right) + {C_4}\cos \left({\sqrt 3 x} \right)+\begin{array}{l} (3{x^4} - 12{x^4}) \end{array} \end{array}\) India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students A solution of the ordinary differential equation \(\frac{{{d^2}y}}{{d{t^2}}} + 5\frac{dy}}{{dt}} + 6y = 0\) is such that \(y\left(0 \right) = 2\) and \(y\left(1 \right) = - \frac{{1 - 3e}}{{{e^3}}}\). The value of \(\frac{dy}}{{dt}}\left(0 \right)\) is _____.Answer (Detailed Solution Below) -3 Given a differential equation, \(\frac{{{d^2}y}}{{d{t^2}}} + 5\frac{dy}}{{dt}} + 6y = 0\) Apply Laplace transform to the above differential equation, ⇒ (s2 + 5s + 6).Y(s) = 0 The characteristic equation will be s2 + 5s + 6 The roots of the equation are - 3, - 2 Solution for the differential equation can be written as \(y\left(t \right) = {C_2}{e^{ - 3t}} + {C_2}{e^{ - 2t}}\) Given y(0) = 2 Therefore \(y\left(0 \right) = {C_1} + {C_2} = 2\) Also given, \(y\left(1 \right) = - \frac{{1 - 3e}}{{{e^3}}}\) Therefore \(y\left(1 \right) = - \left(\frac{{1 - 3e}}{{{e^3}}} \right) = - {e^{ - 3}} + 3{e^{ - 2}} = {C_1}{e^{ - 3}} + {C_2}{e^{ - 2}} \) So \({C_1} = - 1,{C_2} = 3\) So, \(y\left(t \right) = - {e^{ - 3t}} + 3{e^{ - 2t}}\) \(\frac{dy\left(t \right)}{{dt}} = 3{e^{ - 3t}} - 6{e^{ - 2t}},\frac{dy\left(0 \right)}{{dt}} = 3 - 6 = - 3\) India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students The general solution of the differential equation \(\frac{{{d^4}y}}{{d{x^4}}} - 2\frac{{{d^3}y}}{{d{x^3}}} + 2\frac{{{d^2}y}}{{d{x^2}}} - 2\frac{dy}}{{dx}} + y = 0\) is y = (c1 – c2x) ex + c3 cos x + c4 sin xy = (c1 + c2x) ex – c3 cos x + c4 sin xy = (c1 + c2x) ex + c3 cos x + c4 sin xy = (c1 + c2x) ex + c3 cos x – c4 sin x Option 3 : y = (c1 + c2x) ex + c3 cos x + c4 sin x Concept: General equation for DE: \(\frac{{{d^n}y}}{{d{x^n}}} + {k_1}\frac{{{d^{n - 1}}y}}{{d{x^{n - 1}}}} + {k_2}\frac{{{d^{n - 2}}y}}{{d{x^{n - 2}}}} + \ldots {k_n}y = 0\) Then its corresponding Auxiliary equation will be AE: Dn + k1 Dn-1 + ... kn = 0 Then the solution of above DE will be as follows: Roots of Auxiliary Equation Complementary Function m1, m2, m3, ... (real and different roots) \({C_1}{e^{{m_1}x}} + {C_2}{e^{{m_2}x}} + {C_3}{e^{{m_3}x}} + \ldots\) m1, m1, m3, ... (two real and equal roots) \(\left({{C_1} + {C_2}x} \right){e^{{m_1}x}} + {C_3}{e^{{m_3}x}} + \ldots\) m1, m1, m1, m4... (three real and equal roots) \(\left({{C_1} + {C_2}x + {C_3}{x^2}} \right){e^{{m_1}x}} + {C_4}{e^{{m_4}x}} + \ldots\) α + i β, α – i β, m3, ... (a pair of imaginary roots) \({e^{\alpha x}}\left({{C_1}\cos \beta x + {C_2}\sin \beta x} \right) + {C_3}{e^{{m_3}x}} + \ldots\) α ± i β, α ± i β, m5, ... (two pairs of equal imaginary roots) \({e^{\alpha x}}\left({\left({{C_1} + {C_2}x} \right)\cos \beta x + \left({{C_3} + {C_4}x} \right)\sin \beta x} \right) + {C_5}{e^{{m_5}x}} + \ldots\) Calculation: Given: \(\frac{{{d^4}y}}{{d{x^4}}} - \frac{{2{d^3}y}}{{d{x^3}}} + \frac{{2{d^2}y}}{{d{x^2}}} - \frac{{2dy}}{{dx}} + y = 0\) Its Auxillary equations: (D4 – 2D3 + 2D2 – 2D + D) = 0 (D - 1)(D3 - D2 + D - 1) = 0 (D - 1)(D - 1)(D2 + 1) = 0 It roots = 1, 1, i, -i Then it has two equal roots and one pair of imaginary roots. Therefore, the solution is: (c1 + c2x) ex + c3 cos x + c4 sin x = 0 India's #1 Learning Platform Start Complete Exam Preparation Daily Live MasterClasses Practice Question Bank Mock Tests & Quizzes Trusted by 7.2 Crore+ Students The complete solution of the linear differential equation \(\frac{{{d^2}z}}{{d{t^2}}} + \left({p + q} \right)\frac{{dz}}{{dt}} + pqz = 0\) c1 e-pt + c2 e-qtc1 ept - c2 eqtc1 e-pt + c2 eqtc1 ept + c2 eqt Option 1 : c1 e-pt + c2 e-qt For the given differential equation: \(\frac{{{{\rm{d}}^2}{\rm{Z}}}}{{{\rm{d}}{{\rm{t}}^2}}} + \left({{\rm{p}} + {\rm{q}}} \right)\frac{{{\rm{dz}}}}{{{\rm{dt}}}} + {\rm{pqz}} = 0\) \({\rm{D}}_{