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If youve ever wondered how to handle angles that are not so straightforward, youve come to the right place. Half Angle Trig Identities are powerful tools that help us deal with these tricky situations. In this article, well explore the essential half-angle trig identities formulas and provide clear examples to illustrate their practical applications. By the end of this guide, youll have a solid understanding of how to work with half angles in trigonometry and expand your problem-solving skills in this fascinating branch of mathematics. Half angle trig identities, a set of fundamental mathematical relationships used in trigonometry to express trigonometric functions of an angle in terms of functions of half that angle. These identities particularly useful when dealing with complex trigonometric problems, simplifying expressions, and solving equations. The half angle identities derived from the double angle identities and play a crucial role in various branches of mathematics and engineering. The most commonly used half angle identities are:Sine Half Angle Identity: sin(x/2) = [(1 cos(x))/2]Cosine Half Angle Identity: cos(x/2) = [(1 + cos(x))/2]Tangent Half Angle Identity: tan(x/2) = [(1 cos(x))/(1 + cos(x))]Where x is the angle in question. The sign is used to indicate that the identitys value depends on the quadrant in which the angle lies.These identities can employed in a wide range of applications, such as in calculus, physics, engineering, and geometry. They especially valuable when integrating trigonometric functions, simplifying expressions, and solving trigonometric equations. By skillfully utilizing the half angle trig identities, mathematicians and scientists able to tackle complex problems with greater ease and efficiency.Trigonometry Half Angle Formulas and ExamplesTrigonometry half angle formulas play a significant role in solving trigonometric problems that involve angles halved from their original values. These formulas provide a means to express sine, cosine, and tangent functions in terms of half of the original angle, simplifying calculations and manipulations in trigonometric equations.Lets take a look at some examples to illustrate the use of half angle formulas: Example 1: Given that cos(x) = -3/5 and x is in the second quadrant, we want to find cos(x/2). Solution: Since x is in the second quadrant, cos(x/2) positive. Using the cosine half angle identity, we have: cos(x/2) = [(1 + cos(x))/2] cos(x/2) = [(1 3/5)/2] cos(x/2) = [(2/5)/2] cos(x/2) = (1/5) cos(x/2) = 5 / 5Example 2: Given that tan(y) = 4/3 and y is in the fourth quadrant, we want to find tan(y/2). Solution: In the fourth quadrant, tan(y/2) positive. Using the tangent half angle identity, we have: tan(y/2) = [(1 cos(y))/(1 + cos(y))] tan(y/2) = [(1 4/5)/(1 + 4/5)] tan(y/2) = [(1/5)/(9/5)] tan(y/2) = (1/9) tan(y/2) = 1/3These examples demonstrate the application of half angle formulas in finding trigonometric values of angles effectively and efficiently.DoubleAngle and HalfAngle IdentitiesDoubleangle and halfangle identities essential tools in trigonometry that establish relationships between trigonometric functions of angles when those angles either doubled or halved. These identities derived from the sum and difference identities, and they find extensive application in various mathematical and engineering fields.The doubleangle identities are as follows:DoubleAngle Sine Identity: sin(2x) = 2 * sin(x) * cos(x)DoubleAngle Cosine Identity: cos(2x) = cos^2(x) sin^2(x) = 2 * cos^2(x) 1 = 1 2 * sin^2(x)DoubleAngle Tangent Identity: tan(2x) = (2 * tan(x)) / (1 tan^2(x))On the other hand, the halfangle identities derived from the doubleangle identities and areexpressed as:Sine HalfAngle Identity: sin(x/2) = [(1 cos(x))/2]Cosine HalfAngle Identity: cos(x/2) = [(1 + cos(x))/2]Tangent HalfAngle Identity: tan(x/2) = [(1 cos(x))/(1 + cos(x))]The sign in the halfangle identities indicates that the value depends on the quadrant in which the angle lies, similar to the original half angle trig identities.These doubleangle and halfangle identities are instrumental in simplifying trigonometric expressions, solving trigonometric equations, and evaluating integrals. Engineers, physicists, and mathematicians frequently rely on these identities to solve complex problems involving trigonometric functions in various fields. Mastery of these identities is fundamental to advancing ones understanding and proficiency in trigonometry. Visited 221 times, 1 visit(s) today Half-angle identities are trigonometric identities that are used to calculate or simplify half-angle expressions, such as $latex \sin(\frac{\theta}{2})$. These identities can also be used to transform trigonometric expressions with exponents to one without exponents. The half-angle identity of the sine is: $latex \sin(\frac{\theta}{2})=\pm \sqrt{\frac{1-\cos(\theta)}{2}}$ The half-angle identity of the cosine is: $latex \cos(\frac{\theta}{2})=\pm \sqrt{\frac{1+\cos(\theta)}{2}}$ The half-angle identity of the tangent is: $latex \tan(\frac{\theta}{2})=\frac{\sin(\theta)}{1+\cos(\theta)}$$latex =\frac{1-\cos(\theta)}{\sin(\theta)}$ The mean angle identities can be derived using thedouble angle identities. To derive the formula for the identity of half-angle of sines, we start with the double angle identity of cosines: $latex \cos(2\theta)=1-2{{\sin}^2}(\theta)$ If we use the relation $latex \theta=\frac{\alpha}{2}$, we have $latex 2\theta=\alpha$. Substituting these expressions in the identity above, we have: $latex \cos(\alpha)=1-2{{\sin}^2}(\frac{\alpha}{2})$ Now, we solve this expression for $latex \sin(\frac{\alpha}{2})$: $latex \cos(\alpha)=1-2{{\sin}^2}(\frac{\alpha}{2})$ $latex 2{{\sin}^2}(\frac{\alpha}{2})=1-\cos(\alpha)$ $latex {{\sin}^2}(\frac{\alpha}{2})=\frac{1-\cos(\alpha)}{2}$ $latex \sin(\frac{\alpha}{2})=\pm\sqrt{\frac{1-\cos(\alpha)}{2}}$ The sign of $latex \sin(\frac{\alpha}{2})$ depends on the quadrant in which $latex \frac{\alpha}{2}$ is located. If $latex \frac{\alpha}{2}$ is in the first or second quadrant, the formula uses the positive sign, and if $latex \frac{\alpha}{2}$ is in the third or fourth quadrant, the formula uses the negative sign. We use a similar process to find the half-cosine angle identity. Therefore, we start with the double-angle identity of the cosine in the following form: $latex \cos(2\theta)=2{{\cos}^2}(\theta)-1$ After making the substitutions, we get: $latex \cos(\alpha)=2{{\cos}^2}(\frac{\alpha}{2})-1$ Now, we solve for $latex \cos(\frac{\alpha}{2})$: $latex \cos(\alpha)=2{{\cos}^2}(\frac{\alpha}{2})-1$ $latex 2{{\cos}^2}(\frac{\alpha}{2})=1+\cos(\alpha)$ $latex {{\cos}^2}(\frac{\alpha}{2})=\frac{1+\cos(\alpha)}{2}$ $latex \cos(\frac{\alpha}{2})=\pm\sqrt{\frac{1+\cos(\alpha)}{2}}$ In this case, if $latex \frac{\alpha}{2}$ is in the first or fourth quadrant, the formula uses the positive sign and if $latex \frac{\alpha}{2}$ is in the second or third quadrant, the formula uses the negative sign. The following examples are solved using what you have learned about half-angle identities. Study and analyze these examples to understand the process used. Use the half-angle identity of the sine to find the sine value of 15. We use the formula for the half-angle identity of the sine with the given value. Therefore, we have: $latex \sin(\frac{\theta}{2})=\pm\sqrt{\frac{1-\cos(\theta)}{2}}$ $latex \sin(15^{\circ})=\pm\sqrt{\frac{1-\cos(30^{\circ} )}{2}}$ $latex =\pm\sqrt{\frac{1-0.866}{2}}$ $latex =0.259$ We use the positive value since 15 is in the first quadrant. Determine the value of the cosine of 165 using the half-angle identity of the cosine. To use the half-angle identity of cosine, we use the angle $latex \frac{\theta}{2} = 165$. This means that we have $latex \theta = 330$. Therefore, we use the formula with these values: $latex \cos(\theta)=\pm\sqrt{\frac{1+\cos(\theta)}{2}}$ $latex \cos(165^{\circ})=\pm\sqrt{\frac{1+\cos(330^{\circ})}{2}}$ $latex =\pm\sqrt{\frac{1+0.866}{2}}$ $latex =-0.966$ We chose the negative value since the angle 165 is in the second quadrant. Check that the identity $latex 2{{\sin}^2}(\frac{x}{2})+\cos(x)= 1$. We can use the identity of the half-angle of sine to substitute and simplify the expression. By doing this, we have: $latex 2{{\sin}^2}(\frac{x}{2})+\cos(x)$ $latex =2{{(\sqrt{\frac{1-\cos(x)}{2}})}^2}+\cos(x)$ $latex =2(\frac{1-\cos(x)}{2})+\cos(x)$ $latex =1-\cos(x)+\cos(x)$ $latex =1$ After simplifying, we see that the left side of the identity is equal to the right side, so the identity is true. Solve the following practice problems using what you have learned about the half-angle identities of sine, cosine, and tangent. Select an answer and check it to see if you got the correct answer. Interested in learning more about trigonometric identities? Take a look at these pages: Learning Objectives Apply the half-angle identities to expressions, equations and other identities. Use the half-angle identities to find the exact value of trigonometric functions for certain angles. Another use of the cosine double angle identities is to use them in reverse to rewrite a squared sine or cosine in terms of the double angle. Starting with one form of the cosine double angle identity: \[\cos (2\alpha )=2\cos ^{2} (\alpha )-1onumber\]Isolate the cosine squared term\[\cos (2\alpha )+1=2\cos ^{2} (\alpha )onumber\] Add 1\[\cos ^{2} (\alpha )=\dfrac{\cos (2\alpha )+1}{2}onumber\]Divide by 2\[\cos ^{2} (\alpha )=\dfrac{\cos (2\alpha )+1}{2}onumber\] This is called a power reduction identity Exercise \(\PageIndex{1}\) Use another form of the cosine double angle identity to prove the identity \(\sin ^{2} (\alpha )=\dfrac{1-\cos (2\alpha )}{2}\). Answer \[\begin{array}{l} {\dfrac{1-\cos (2\alpha )}{2} } \\ {\dfrac{1-\left(\cos ^{2} (\alpha )-\sin ^{2} (\alpha )\right)}{2} } \\ {\dfrac{1-\cos ^{2} (\alpha )+\sin ^{2} (\alpha )}{2} } \\ {\dfrac{\sin ^{2} (\alpha )+\sin ^{2} (\alpha )}{2} } \\ {\dfrac{2\sin ^{2} (\alpha )}{2} =\sin ^{2} (\alpha )} \end{array}onumber\] The cosine double angle identities can also be used in reverse for evaluating angles that are half of a common angle. Building from our formula \(\cos ^{2} (\alpha )=\dfrac{\cos (2\alpha )+1}{2}\), if we let \(\theta =2\alpha\), then \(\alpha =\dfrac{\theta }{2}\) this identity becomes \(\cos ^{2} \left(\dfrac{\theta }{2} \right)=\dfrac{\cos (\theta )+1}{2}\). Taking the square root, we obtain \[\cos \left(\dfrac{\theta }{2} \right)=\pm \sqrt{\dfrac{\cos (\theta )+1}{2} }onumber\]where the sign is determined by the quadrant. This is called a half-angle identity. Exercise \(\PageIndex{2}\) Use your results from the last Try it Now to prove the identity \(\sin \left(\dfrac{\theta }{2} \right)=\pm \sqrt{\dfrac{1-\cos (\theta )}{2} }\). Answer \[\begin{array}{l} {\sin ^{2} (\alpha )=\dfrac{1-\cos (2\alpha )}{2} } \\ {\sin (\alpha )=\pm \sqrt{\dfrac{1-\cos (2\alpha )}{2} } } \\ {\alpha =\dfrac{\theta }{2} } \\ {\sin \left(\dfrac{\theta }{2} \right)=\pm \sqrt{\dfrac{1-\cos \left(2\left(\dfrac{\theta }{2} \right)\right)}{2} } } \\ {\sin \left(\dfrac{\theta }{2} \right)=\pm \sqrt{\dfrac{1-\cos (\theta )}{2} } } \end{array}onumber\] IDENTITIES Half-Angle Identities \[\cos \left(\dfrac{\theta }{2} \right)=\pm \sqrt{\dfrac{\cos (\theta )+1}{2} }\] \[\sin \left(\dfrac{\theta }{2} \right)=\pm \sqrt{\dfrac{1-\cos (\theta )}{2} }\] Power Reduction Identities \[\cos ^{2} (\alpha )=\dfrac{\cos (2\alpha )+1}{2}\] \[\sin ^{2} (\alpha )=\dfrac{1-\cos (2\alpha )}{2}\] Since these identities are easy to derive from the double-angle identities, the power reduction and half-angle identities are not ones you should need to memorize separately. Example \(\PageIndex{1}\) Rewrite \(\cos ^{4} (x)\) without any powers. Solution \[\cos ^{4} (x)=\left(\cos ^{2} (x)\right)^{2}onumber\]Using the power reduction formula\[=\left(\dfrac{\cos (2x)+1}{2} \right)^{2}onumber\]Square the numerator and denominator\[=\dfrac{\left(\cos (2x)+1\right)^{2} }{4}onumber\]Expand the numerator\[=\dfrac{\cos ^{2} (2x)+2\cos (2x)+1}{4}onumber\]Split apart the fraction\[=\dfrac{\cos ^{2} (2x)}{4} +\dfrac{2\cos (2x)}{4} +\dfrac{1}{4}onumber\]Apply the formula above to \(\cos ^{2} (2x)\)\[\cos ^{2} (2x)=\dfrac{\cos (2\cdot 2x)+1}{2}onumber\]\[=\dfrac{\left(\dfrac{\cos (4x)+1}{2} \right)}{4} +\dfrac{2\cos (2x)}{4} +\dfrac{1}{4}onumber\]Simplify\[=\dfrac{\cos (4x)}{8} +\dfrac{1}{8} +\dfrac{1}{2} \cos (2x)+\dfrac{1}{4}onumber\]Combine the constants\[=\dfrac{\cos (4x)}{8} +\dfrac{1}{2} \cos (2x)+\dfrac{3}{8}onumber\]Example \(\PageIndex{2}\) Find an exact value for \(\cos \left(15{}^\circ \right)\). Solution Since 15 degrees is half of 30 degrees, we can use our result from above: \[\cos (15{}^\circ )=\cos \left(\dfrac{30{}^\circ }{2} \right)=\pm \sqrt{\dfrac{\cos (30{}^\circ )+1}{2} }onumber\] We can evaluate the cosine. Since 15 degrees is in the first quadrant, we need the positive result. \[\sqrt{\dfrac{\cos (30{}^\circ )+1}{2} } =\sqrt{\dfrac{\dfrac{\sqrt{3} }{2} +1}{2} }onumber\]\[=\sqrt{\dfrac{\sqrt{3} }{4} +\dfrac{1}{2} }onumber\]Exercise \(\PageIndex{3}\) If \(\csc \left(x\right)=7\) and \(90{}^\circ